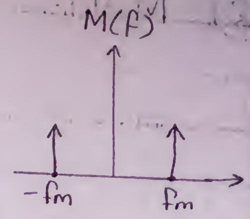
Summary of AM rules

① Single tone modulation

means $m(t)$ is only one freq. f_m

- $m(t) = A_m \cos(2\pi f_m t)$, $c(t) = A_c \cos(2\pi f_c t)$ →



one freq.

- $S(t) = A_c\,(1 + \underbrace{K_a . A_m}_{\mu} \cos(2\pi f_m t)) \cos(2\pi f_c t)$

- $\mu = \dfrac{A_m}{A_c} = K_a . A_m$

$$S(t) = \underbrace{A_c \cos(2\pi f_c t)}_{\text{Carrier}} + \left.\frac{A_c\,\mu}{2}\Big[\cos(2\pi(f_c - f_m)t) + \cos(2\pi(f_c + f_m)t)\Big]\right\} \begin{matrix}\text{DSB}\\ = \\ m(t) * c(t)\end{matrix}$$

- Average power

$$P_{avg.} \xrightarrow{} R_L = 1 \xrightarrow{=} \frac{V_{Peak}^2}{2} = \frac{1}{T_0}\int_0^{T_0} g^2(t)\,dt$$

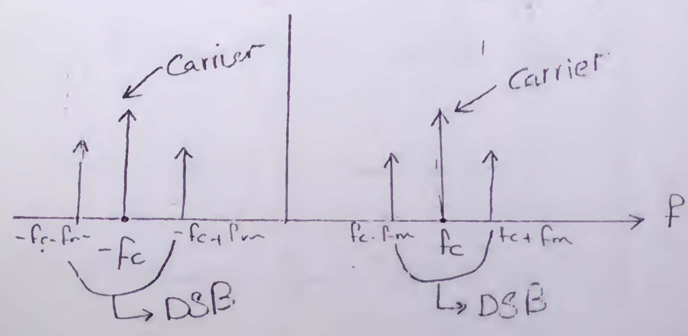
$$P_c = \frac{A_c^2}{2} \quad , \quad P_{SB} = \frac{A_c^2 \mu^2}{8} \quad , \quad P_{DSB} = \frac{A_c^2 \mu^2}{4}$$

$$P_t = P_c + P_{DSB} = \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right]$$

$$\eta = \frac{P_{DSB}}{P_t}\,\% = \frac{A_c^2 \mu^2/4}{A_c^2/2 + A_c^2 \mu^2/4}\,\% = \frac{\mu^2}{2 + \mu^2}\,\%$$

- Peak Power $= \dfrac{V_{Peak}^2}{R_L} = I_{Peak}^2 . R_L$.

$$P_{c_{Peak}} = \frac{A_c^2}{R_L} \quad , \quad P_{DSB_{Peak}} = \frac{A_c^2 \mu^2}{2 R_L}$$



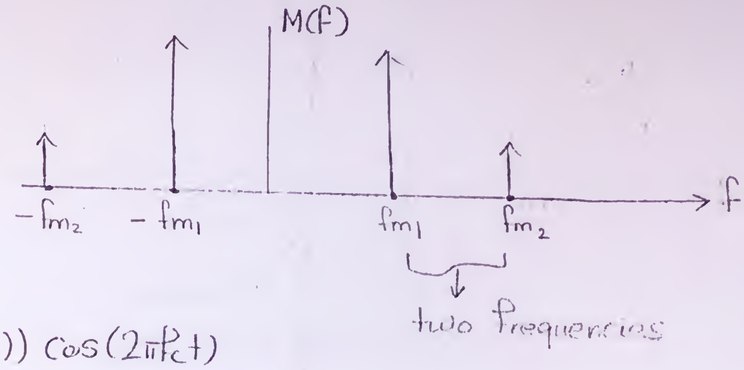
② Multitone Modulation

$$m(t) = A_{m_1}\cos(2\pi f_{m_1} t) + A_{m_2}\cos(2\pi f_{m_2} t)$$

سواء cos

أو sin

هنا الـ message أكثر من تردد فبيسمى بالـ Multitone

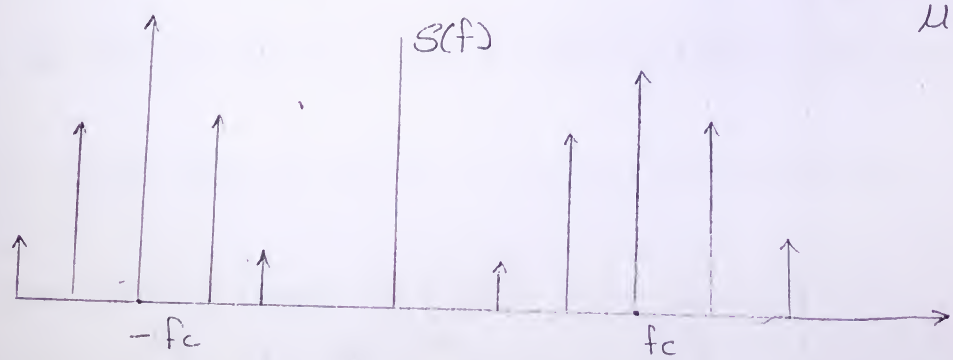


$$S(t) = A_c\,(1 + K_a \cdot m(t))\cos(2\pi f_c t)$$

$$S(t) = A_c\left[1 + \mu_1\cos(2\pi f_{m_1} t) + \mu_2\cos(2\pi f_{m_2} t)\right]\cos(2\pi f_c t)$$

- $\mu_t = \sqrt{\mu_1^2 + \mu_2^2}$ ↳ Total

- $P_{DSB} = \dfrac{A_c^2\,\mu_t^2}{4}$ ← مثل ← . كل باقي القوانين كما هي لكن نعوض بدل μ بالـ μ_t



نفس طريقة الرسم : 1) δ عند f_c و $-f_c$

2) عمل shift لـ $M(f)$ حول f_c و $-f_c$

Remember

$$A\cos(2\pi f_c t) \rightleftharpoons \frac{A}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right]$$

$$A\sin(2\pi f_c t) \rightleftharpoons \frac{A}{2j}\left[\delta(f-f_c) - \delta(f+f_c)\right].$$

Sheet #3 — AM Sheet

I) An AM signal has the form

$$u(t) = [20 + 2\cos(3000\pi t) + 10\cos(6000\pi t)] \cdot \cos(2\pi f_c t)$$

where $f_c = 10^5$ Hz.

1. Sketch the voltage spectrum of $u(t)$.

$U(f)$? → draw

$$u(t) = 20\cos(2\pi f_c t) + 2\cos(\underbrace{3000\pi t}_{2 * 1500 \cdot \pi \cdot t}) \cdot \cos(2\pi \cdot 10^5 t)$$

$$+ 10\cos(6000\pi t) \cdot \cos(2\pi \cdot 10^5 t)$$

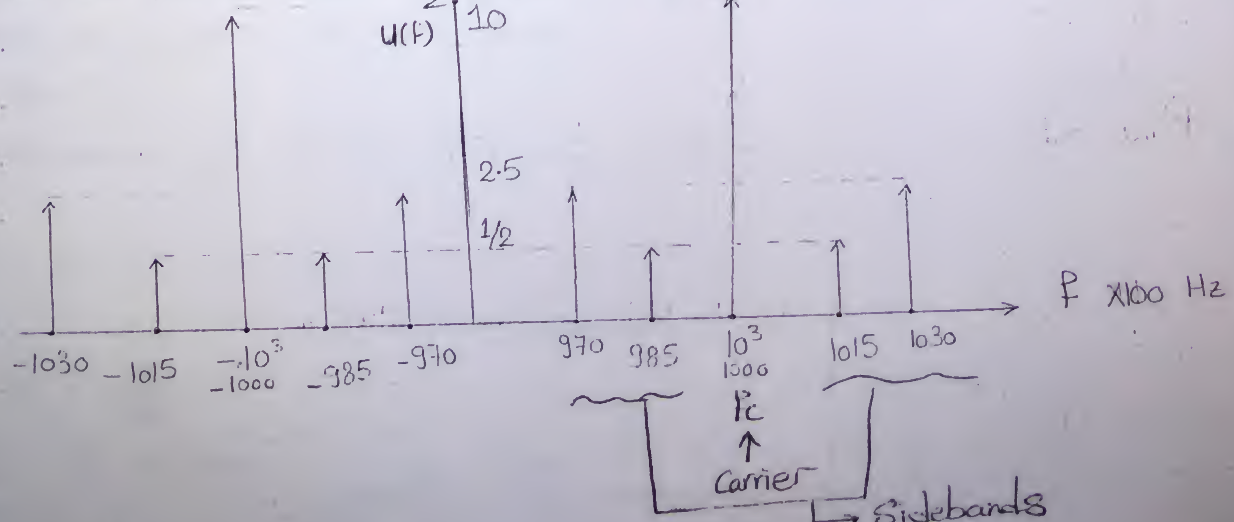
$$= 20\cos(2\pi f_c t) + \frac{2}{2}\left[\cos(2\pi(10^5 - 1500)t) + \cos(2\pi(10^5 + 1500)t)\right]$$

$$+ \frac{10}{2}\left[\cos(2\pi(\overset{f_c}{10^5} - 3000)t) + \cos(2\pi(\overset{f_c}{10^5} + 3000)t)\right]$$

$$= 20\cos(2\pi \cdot 10^5 t) + \cos(2\pi(98.5 \times 10^3)t) + \cos(2\pi(101.5 \times 10^3)t)$$

$$+ 5\cos(2\pi(97 \times 10^3)t) + 5\cos(2\pi(103 \times 10^3)t).$$

$$A\cos(2\pi f_o t) \rightleftharpoons \frac{A}{2}\left[\delta(f - f_o) + \delta(f + f_o)\right]$$



2. Determine the Power in each of the frequency Components.

$$\boxed{\text{Remember that for } A\cos\theta \longrightarrow P_{avg.} = \frac{A^2}{2}}$$

هاشوف كل δ في ال Spectrum ال cosine بتاعتها فين وأربع ال Peak وأقسمها على اتنين.

- $f = 10^5 \longrightarrow P_{avg.} = \frac{20^2}{2} = 200\ W$
- $f = 98.5 \times 10^3$ $(f_c - 1500)$ and 101.5×10^3 $(f_c + 1500)$ $\longrightarrow P_{avg.} = \frac{1}{2}\ W$
- $f = 97 \times 10^3$ and $103 \times 10^3 \longrightarrow P_{avg.} = \frac{5^2}{2} = 12.5\ W$

3. Determine the modulation index

$$u(t) = 20\left[1 + \underbrace{0.1}_{\mu_1}\cos(3000\pi t) + \underbrace{0.5}_{\mu_2}\cos(6000\pi t)\right]\cos(2\pi f_c t)$$

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = 0.509$$ for multitone $m(t)$

4. Determine the Power in the sidebands, the total Power and the ratio of the sidebands power to the total Power.

from 2.

$$P_{sidebands} = \frac{1}{2} + 12.5 + 12.5 + \frac{1}{2} = 26\ W.$$

$$P_c = 200\ W.$$

$$P_t = P_c + P_{sidebands} = 226\ W.$$

$$Ratio = \frac{26}{226}$$

Note: for multitone use $P_{DSB} = A_c^2 \cdot \mu_t^2 / 4 = P_c \cdot \frac{\mu_t^2}{2}$

(2) An AM signal is generated by modulating the carrier $f_c = 800$ KHz by the signal

$$m(t) = \sin(2000\pi t) + 5\cos(4000\pi t)$$

The AM signal

$$u(t) = 100\,[1 + m(t)]\cos(2\pi f_c t)$$

is fed to a $50\,\Omega$ load

1. Determine & sketch the spectrum of the AM signal.

$$u(t) = 100\,[1 + \overset{\mu_1}{1}\sin(2\pi(1000)t) + \overset{\mu_2}{5}\cos(2\pi(2000)t)]\cos(2\pi f_c t)$$

$$= 100\cos(2\pi f_c t) + \frac{100}{2}[\sin(2\pi(1000 - f_c)t) + \sin(2\pi(1000+f_c)t)]$$

$$+ \frac{500}{2}[\cos(2\pi(2000 - f_c)t) + \cos(2\pi(2000+f_c)t)]$$

$$= 100\cos(2\pi f_c t) - 50\sin(2\pi(f_c - 1000)t) + 50\sin(2\pi(f_c+1000)t)$$

$$+ 250\cos(2\pi(f_c - 2000)t) + 250\cos(2\pi(f_c+2000)t).$$

$$\sin x \cdot \cos y = \frac{1}{2}[\sin(x-y) + \sin(x+y)]$$

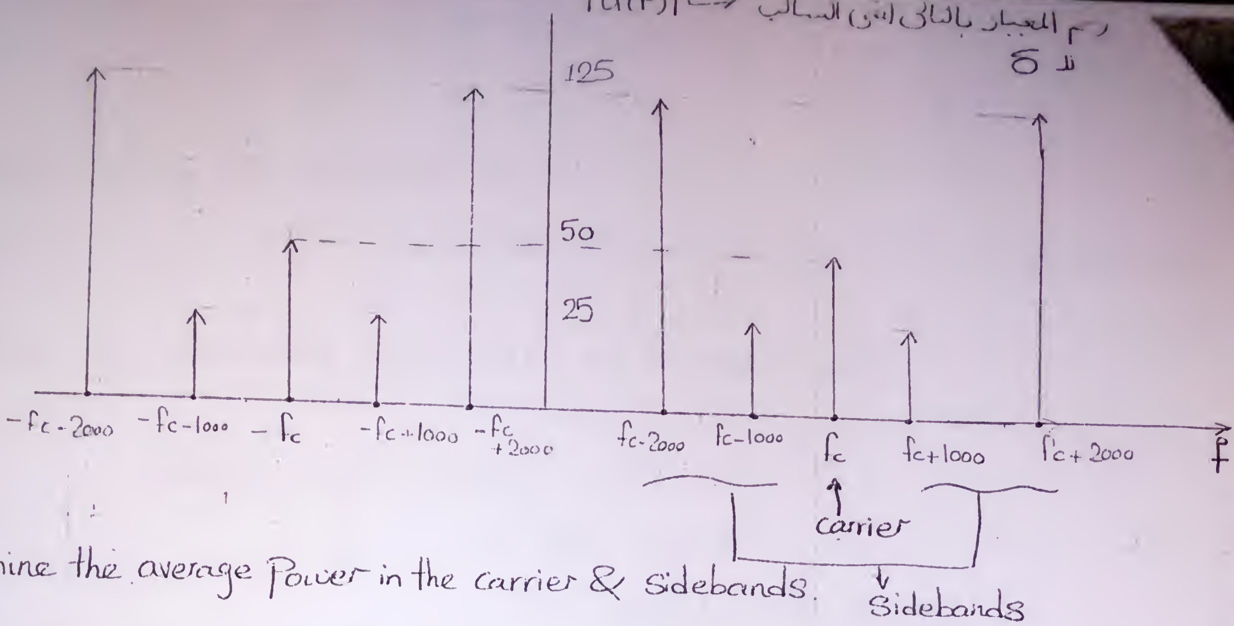
$$A\cos 2\pi f_0 t \rightleftharpoons \frac{A}{2}[\delta(f-f_c) + \delta(f+f_c)]$$

$$A\sin 2\pi f_0 t \rightleftharpoons \frac{A}{2j}[\delta(f-f_c) - \delta(f+f_c)]$$

$$U(f) = 50[\delta(f-f_c) + \delta(f+f_c)] - \frac{25}{j}[\delta(f-(f_c+1000)) - \delta(f+(f_c+1000))]$$

$$+ \frac{25}{j}[\delta(f-(f_c+1000)) - \delta(f+(f_c+1000))] + 125[\delta(f-(f_c-2000))$$

$$+ \delta(f+(f_c-2000))] + 125[\delta(f-(f_c+2000)) + \delta(f+(f_c+2000))].$$



2. Determine the average Power in the carrier & Sidebands.

$$P_c = \frac{100^2}{2} = 5000 \, W$$

$$P_{sidebands} = \frac{50^2}{2} + \frac{250^2}{2} + \frac{50^2}{2} + \frac{250^2}{2} = 65000 \, W$$

3. What is the modulation index ?

$\mu_1 = 1$

$\mu_2 = 5$

$\mu_t = \sqrt{1 + 5^2} = 5.099$.

4. What is the Peak Power delivered to the load ?

(3) The output of an AM modulator is

$$u(t) = 5\cos(1800\pi t) + 20\cos(2000\pi t) + 5\cos(2200\pi t)$$

1. Determine the modulating signal m(t) and the carrier c(t).

$$S(t) = \underset{f_c}{c(t)} + k_a \underbrace{m(t) * c(t)}_{\text{if both cosines}}$$

$$\cos(\underset{f_c - f_m}{\text{الفرق}}) + \cos(\underset{f_c + f_m}{\text{المجموع}})$$

$\therefore$ $\boxed{20\cos(2000\pi t) \rightarrow \text{Carrier}}$

$5\cos(1800\pi t) + 5\cos(2200\pi t) \rightarrow k_a m(t) \cdot c(t)$

$$u(t) = 20\left[1 + \frac{1}{2}\cos(200\pi t)\right]\cos(2000\pi t)$$

$* \; 20\cos(2000\pi t) \cdot k_a \cdot m(t) = 5\left[\cos(1800\pi t) + \cos(2200\pi t)\right]$

$\frac{1}{2}\cos(200\pi t)$ $\qquad$ $\frac{20 * A_m * k_a}{2} \xrightarrow{k_a A_m} A_m = 1/2$

$\therefore$ $\boxed{m(t) = \cos(200\pi t)}$

2. Determine the modulation index.

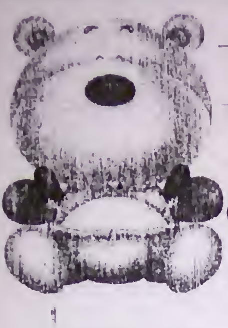
$$\mu = k_a \cdot A_m = \frac{1}{2}$$

3. Determine the ratio of the power in the sidebands to the power in the carrier.

$$P_c = \frac{20^2}{2} = 200 \text{ W}$$

$$P_{sideband} = \frac{A_c^2 \cdot \mu^2}{4} = \frac{(20 * 1/2)^2}{4} = 25 \text{ W.} \quad \text{or} \quad \frac{5^2}{2} + \frac{5^2}{2}$$

$$\therefore \frac{P_{sideband}}{P_c} = \frac{25}{200} = \frac{1}{8}.$$



Sheet # 3

(1) audio signal $m(t) = 15\cos(2\pi.1500\,t) \rightarrow A_m = 15 \quad f_m = 1500\ Hz$

$c(t) = 60\cos(2\pi.100000\,t) \rightarrow A_c = 60 \quad f_c = 10^5\ Hz$

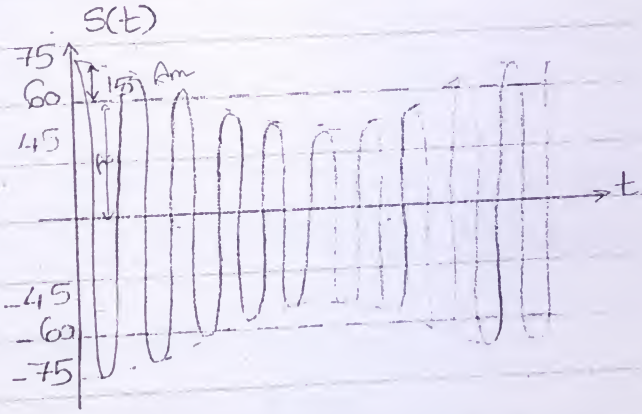
a) AM equation

$$S(t) = A_c\,(1 + \underbrace{K_a . A_m}_{\mu} \cos(2\pi f_m t))\cos(2\pi f_c t)$$

$$\mu = \frac{A_m}{A_c} = \frac{15}{60} = 0.25 \quad \text{or} \quad \mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{(A_c + A_m) - (A_c - A_m)}{(A_c + A_m) + (A_c - A_m)}$$

$$S(t) = 60 . (1 + 0.25\cos(2\pi.1500t))\cos(2\pi.100000\,t)$$

b) Sketch the AM wave.



$\mu < 1 \rightarrow$ under modulation

c) Modulation (index / Factor) μ , Percent modulation

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{75 - 45}{75 + 45} = 0.25$$

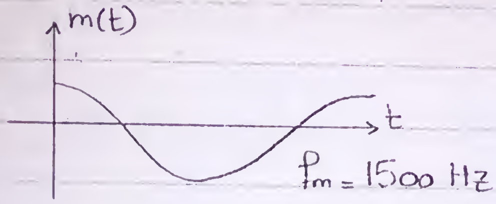
$$\mu\% = 0.25 * 100 = 25\,\%$$

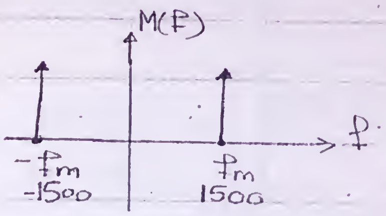
$c(t)(1+k_a\, m(t))$

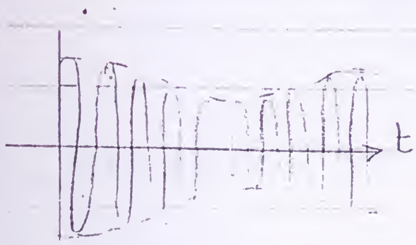
$c(t) + k_a \cdot m(t) \cdot c(t)$

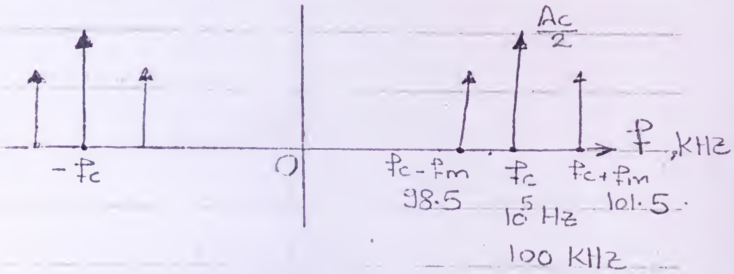
d) AM spectrum

$m(t) \longrightarrow M(f)$









$B.W = 2f_m = 3\,KHz$

f) $f_u = f_c + f_m = 100K + 1.5\,KHz = 101.5\,KHz$

$f_L = f_c - f_m = 100K - 1.5\,KHz = 98.5\,KHz$

Note: B.W. of $m(t) = 1500\,Hz$

" " $S(t) = 3\,KHz$

(2) $P_t = 1000\,w$

$\mu\% = 100\% = 1$

* $P_c = ?$ ** P sideband = ?

* $P_t = P_c \cdot [1 + \frac{\mu^2}{2}] \rightarrow P_c = \frac{1000}{(1+1/2)} = 666.66$ watt

* $P_{DSB} = P_t - P_c = 1000 - 666.66 = 333.33$ watt

$$P_{L.S.B} = P_{U.S.B} = \frac{P_{D.S.B}}{2} = 166.67 \text{ watts}.$$

(3) Repeat for $\mu = 80\% = 0.8$

$$P_t = P_c\left(1 + \frac{\mu^2}{2}\right) \longrightarrow P_c = \frac{1000}{\left(1 + \frac{0.8^2}{2}\right)} = 757.57 \text{ watts}$$

$$P_{L.S.B} = P_{U.S.B} = \frac{P_{DSB}}{2} = \frac{1000 - 757.57}{2} = 121.21 \text{ watts}$$

(4) $P_c = 5K \text{ watt} = 5000 \text{ watts}$ $\qquad \mu = 0.75 = 75\%$

a) $P_t = ?$

$$P_t = P_c\left(1 + \frac{\mu^2}{2}\right) = 5000\left(1 + \frac{0.75^2}{2}\right) = 6406.25 \text{ watts}$$

b) $P_{L.S.B}$ و $P_{U.S.B}$, η

$$P_{DSB} = P_t - P_c = 6406.25 - 5000 = 1406.25 \text{ watts}$$

• $P_{L.S.B} = P_{USB} = \frac{P_{DSB}}{2} = 703.125 \text{ watts}$

• $\eta = \frac{P_{DSB}}{P_t}\% = \frac{1406.25}{6406.25}\% = 21.95\%$

c) Comment

نلاحظ أن الكفاءة منخفضة جداً وذلك لأن معظم الباور ضائعة في الـ carrier الذي
لا تحتوي على أي معلومة

⑤ $\mu\% = ?$ Percent modulation

$P_c = 8\ kW$

$P_{SB} = 2\ kW$

$P_{DSB} = 2 * 2kW = 4\ kW$

$P_t = P_c + P_{DSB} = 8 + 4 = 12\ kW$

$P_t = P_c\left(1 + \frac{\mu^2}{2}\right) \longrightarrow 1 + \frac{\mu^2}{2} = \frac{P_t}{P_c} = \frac{\cancel{12}^{\,3}}{\cancel{8}_{\,2}}$

$\frac{\mu^2}{2} = \frac{3}{2} - 1 \longrightarrow \boxed{\mu = 1}$

$\therefore \mu\% = 100\%$

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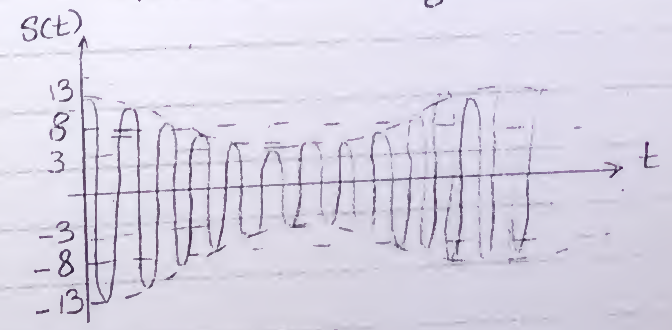
⑥ $m(t) = 5\cos(2\pi 500 t) \longrightarrow A_m = 5,\ f_m = 500$

$P_c = 32$ watt

$K_a = 0.125 = \frac{1}{A_c} \rightarrow A_c = \frac{1}{0.125} = 8 \quad \boxed{A_c = 8}$

a) Sketch AM wave

$\mu = K_a \cdot A_m = \frac{1}{8} * 5 = 0.625$



b) Write AM equation

$S(t) = A_c(1 + k_a \cdot m(t)) \cdot \cos(2\pi f_c t)$

$S(t) = 8(1 + 0.625\cos(2\pi \cdot 500 t))\cos(2\pi f_c t)$

c) $P_t = ?$ $\eta = ?$ Comment

$$P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$$

$$= 32\left(1 + \frac{0.625^2}{2}\right) = 38.25 \text{ watts}$$

$$P_{DSB} = P_t - P_c = 38.25 - 32 = 6.25 \text{ watts}$$

$$\eta = \frac{P_{DSB}}{P_t} \% = 16.3 \%$$

we notice that η is low due to the unuseful Power wasted in the Carrier, $\eta = \frac{P_{useful}}{P_t} \% = \frac{P_{useful}}{P_c + P_{useful}} \%$

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d) B.W of $m(t) = f_m = 500$ Hz
B.W. of AM $= 2f_m = 1000$ Hz
